Diploma Programme subject outline—Group 5: mathematics			
School name	International School of Lausanne	**School code**	001079
Name of the DP subject	Mathematics		
Level	☐ Higher Level	☑ Standard Level	

Course Background

Mathematics is in a sense an international language, and, apart from slightly differing notation, mathematicians from around the world can communicate within their field. Mathematics transcends politics, religion and nationality, yet throughout history great civilizations owe their success in part to their mathematicians being able to create and maintain complex social and architectural structures.

Mathematics Standard Level caters for students with a basic background in mathematics who are competent in the skills they have encountered so far. The majority of SL students will be expecting to need a sound mathematical background for university courses such as chemistry, psychology and business. Others may choose this option simply because they have a strong interest in mathematics. The course introduces students to important concepts through the development of techniques, in a comprehensible and coherent way, rather than insisting on mathematical rigor.

The internally assessed component, the exploration, offers students the opportunity for developing independence in their mathematical learning. Students are encouraged to take a considered approach to various mathematical activities and to explore different mathematical ideas. The exploration also allows students to work without the time constraints of a written examination and to develop the skills they need for communicating mathematical ideas.

Students needing a higher degree of mathematical content should consider Mathematics HL. Students wishing to study a less rigorous mathematics course would be advised to opt for the Mathematical Studies SL course.

Assessment in IB Diploma Mathematics SL

Assessment Components – Higher Level	Weighting (%)
External assessment (3 hours)	
Paper 1 (1.5 hours) This paper consists of section A, short-response questions, and section B, extended-response questions. Questions of varying levels of difficulty and length are set and knowledge of **all** topics is required for this paper. The emphasis is on problem-solving. Students are not permitted access to any calculator on this paper.	40%
Paper 2 (1.5 hours) This paper consists of section A, short-response questions, and section B, extended-response questions. Questions of varying levels of difficulty and length are set and knowledge of all topics is required for this paper. The emphasis is on problem-solving. A GDC is required for this paper, but not every question will necessarily require its use.	40%
Internal Assessment	
An individual exploration.	20%

The Nature and Timing of Internal Assessment in IB Diploma Mathematics SL

The exploration is a piece of written work that involves investigating an area of mathematics. It enables students to demonstrate the application of their mathematical skills and knowledge, and to pursue their personal interests, without the time limitations and other constraints that are associated with written examinations. The product is a short report written by the student based on a topic chosen by him or her, and it should focus on the mathematics of that particular area. The emphasis is on mathematical communication (including formulae, diagrams, graphs and so on), with accompanying commentary, good mathematical writing and thoughtful reflection.

Internal Assessment in IB Diploma mathematics SL is undertaken in the first term of Year 13. A full first draft is due in mid-November, and the final version in the second week of December (see calendar of deadlines).

Topic/unit (as identified in the IB Diploma subject guide)	Content	Clarification	Allocated time	Assessment	Resources
Algebra **1.1 Sequences and series** **1.3 The binomial theorem**	Arithmetic sequences and series; sum of finite arithmetic series; geometric sequences and series; sum of finite and infinite geometric series. Sigma notation. Applications Expansion of $(a+b)^n$, $n \in \mathbb{N}$. Calculation of binomial coefficients using Pascal's triangle and $\binom{n}{r}$	Technology may be used to generate and display sequences in several ways. Link to 2.6, exponential functions Examples include compound interest and population growth. Counting principles may be used in the development of the theorem. $\binom{n}{r}$should be found using both the formula and technology. Link to 5.8, binomial distribution. Not required: formal treatment of permutations and formula for nPr .	August-September 2 weeks		Buchanan, Fensom, Kemp "Mathematics Standard Level, Course companion" IB Questionbank

Topic/unit (as identified in the IB Diploma subject guide)	Content	Clarification	Allocated time	Assessment	Resources
Circular functions and trigonometry					
3.1 Radian measure	The circle: radian measure of angles; length of an arc; area of a sector.	Radian measure may be expressed as exact multiples of π , or decimals.	September 3 weeks	Assignment	Buchanan, Fensom, Kemp "Mathematics Standard Level, Course companion"- Oxford
3.2 Unit circle	Definition of cosθ and sinθ in terms of the unit circle. Definition of tanθ as $\frac{sin\theta}{cos\theta}$. Exact values of trigonometric ratios of $0, \frac{\pi}{6}, \frac{\pi}{4}. \frac{\pi}{3}, \frac{\pi}{2}$ and their multiples.	The equation of a straight line through the origin is y x = tanθ .		Test on "Algebra and Trigonometry"	
3.6 Solution of triangles	The cosine rule. The sine rule, including the ambiguous case. Area of a triangle. Applications.	Pythagoras' theorem is a special case of the cosine rule. Link with 4.2, scalar product. Examples include navigation, problems in two and three dimensions, including angles of elevation and depression.			IB Questionbank

Topic/unit (as identified in the IB Diploma subject guide)	Content	Clarification	Allocated time	Assessment	Resources
Functions and equations					
2.1 Concept of function	Domain, range; image (value). Composite functions. Identity function. Inverse function.	A graph is helpful in visualizing the range. Students will only be asked to find the inverse of a one-to-one function.	October 4 weeks	Assignment Test	Buchanan, Fensom, Kemp "Mathematics Standard Level, Course companion"- Oxford
2.2 Solving functions graphically	The graph of a function; its equation y =*f(x).* Function graphing skills. Investigation of key features of graphs, such as maximum and minimum values, intercepts, horizontal and vertical asymptotes, symmetry, and consideration of domain and range. Use of technology to graph a variety of functions, including ones not specifically mentioned. The graph of $y=f^{-1}(x)$ as the reflection in the line y =x of the graph of y =*f(x)*.	Note the difference in the command terms "draw" and "sketch". An analytic approach is also expected for simple functions, including all those listed under topic 2. Link to 6.3, local maximum and minimum points.			IB Questionbank
2.3 Transformation of graphs	Translations. Reflections (in both axes). Vertical stretch with scale factor p. Stretch in the x-direction with scale factor $\frac{1}{q}$. Composite transformations.	Technology should be used to investigate these transformations. Translation by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$denotes			

Topic/unit (as identified in the IB Diploma subject guide)	Content	Clarification	Allocated time	Assessment	Resources
		horizontal shift of 3 units to the right, and vertical shift of 2 down.			
Circular functions and trigonometry **3.4 The circular functions sin x , cos x and tan x** **3.5 Trigonometric equations** **3.3 Double angles**	Their domains and ranges; amplitude, their periodic nature; and their graphs. Composite functions of the form *f(x)=asin(b(x-c))+d* Solving trigonometric equations in a finite interval, both graphically and analytically. Equations leading to quadratic equations in sinx , cosx or tanx . The Pythagorean identity $\cos^2 x + \sin^2 x = 1$. Double angle identities for sine and cosine. Relationship between trigonometric ratios.	Link to 2.3, transformation of graphs. Examples include height of tide, motion of a Ferris wheel. Not required: the general solution of trigonometric equations Simple geometrical diagrams and/or technology may be used to illustrate the double angle formulae (and other trigonometric identities).	November 3 weeks	Assignment Test: Paper 1: Non GDC Paper 2:GDC	Buchanan, Fensom, Kemp "Mathematics Standard Level, Course companion"-Oxford Smedley and Wiseman "Mathematics Standard Level for the IB Diploma"-Oxford IB Questionbank
Algebra **1.2 Exponents and logarithms**	Elementary treatment of exponents and logarithms. Laws of exponents; laws of logarithms. Change of base.	 Link to 2.6, logarithmic functions.	November 2 weeks		

Topic/unit (as identified in the IB Diploma subject guide)	Content	Clarification	Allocated time	Assessment	Resources
2.6 The exponential and logarithmic functions **2.7 Exponential equations**	Exponential functions and their graphs. Logarithmic functions and their graphs. Relationships between these functions. Solving exponential equations.	Links to 1.1, geometric sequences; 1.2, laws of exponents and logarithms; 2.1, inverse functions; 2.2, graphs of inverses; and 6.1, limits. Link to 1.2, exponents and logarithms.			Buchanan, Fensom, Kemp "Mathematics Standard Level, Course companion"- Oxford IB Questionbank
Functions and equations **2.7 Quadratic equations**	Solving equations, both graphically and analytically. Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach. The quadratic formula. The discriminant and the nature of the roots. The quadratic function $f(x)=ax^2+bx+c$: its graph, y-intercept (0, c) . Axis of symmetry.	Solutions may be referred to as roots of equations or zeros of functions. Link to 2.2, function graphing skills; and 2.3-2.6, equations involving specific functions. Candidates are expected to be able to change	August-December 3 weeks		Buchanan, Fensom, Kemp "Mathematics Standard Level, Course companion" IB Questionbank

Topic/unit (as identified in the IB Diploma subject guide)	Content	Clarification	Allocated time	Assessment	Resources
2.4 The quadratic function, completing the Square	The form $f(x)=a(x-p)(x-q)$, x-intercepts (p , 0) and (q , 0) . The form $f(x)=a(x-h)^2+k$, vertex (h,k) .	from one form to another. Links to 2.3, transformations; 2.7, quadratic equations.			
Internal Examinations	Exam preparation, internal examinations and feedback.		1 week	January Examinations	
Calculus **6.1 Differentiation theory**	Informal ideas of limit and convergence. Limit notation. Definition of derivative from first principles. Derivative interpreted as gradient function and as rate of change.	Technology should be used to explore ideas of limits, numerically and graphically. Links to 1.1, infinite geometric series; 2.5–2.7, rational and exponential functions, and asymptotes. Use of this definition for derivatives of simple polynomial functions only. Technology could be used to illustrate other derivatives. Link to 1.3, binomial theorem. Use of both forms of notation for the first derivative.	February-April 8 weeks	Assignment Test 1 on differentiation theory: Paper1: non GDC Paper 2:GDC	Buchanan, Fensom, Kemp “Mathematics Standard Level, Course companion”-Oxford Smedley and Wiseman “Mathematics Standard Level for the IB Diploma”-Oxford

Topic/unit (as identified in the IB Diploma subject guide)	Content	Clarification	Allocated time	Assessment	Resources
	Tangents and normals, and their equations.	Identifying intervals on which functions are increasing or decreasing. Use of both analytic approaches and technology. Technology can be used to explore graphs and their derivatives.			IB Questionbank
6.2 Differentiation rules	Derivative of x^n ($n \in \mathbb{Q}$), e^x and ln x. Differentiation of a sum and a real multiple of these functions. The chain rule for composite functions. The product and quotient rules.	Not required: analytic methods of calculating limits.		Assignment	
	The second derivative. Extension to higher derivatives. Local maximum and minimum points.	Link to 2.1, composition of functions. Technology may be used to investigate the chain rule. Use of both forms of notation.		Test 2 on differentiation rules: Paper1: non GDC Paper 2:GDC	
6.3 Differentiation	Testing for maximum or minimum.	Using change of sign of the first derivative and using sign of the second derivative.			

Topic/unit (as identified in the IB Diploma subject guide)	Content	Clarification	Allocated time	Assessment	Resources
	Points of inflexion with zero and non-zero gradients. Graphical behaviour of functions, including the relationship between the graphs of f , f ′ and f ′′. Optimization. Applications	Use of the terms "concave-up" for f ′′(x)>0 , and "concave-down" for f ′′(x)< 0. At a point of inflexion , f ′′ (x) = 0 and changes sign (concavity change). f ′′(x) =0 is not a sufficient condition for a point of inflexion. Both "global" (for large x) and "local" behaviour. Technology can display the graph of a derivative without explicitly finding an expression for the derivative. Use of the first or second derivative test to justify maximum and/or minimum values. Examples include profit, area, volume. Link to 2.2, graphing functions. Not required: points of inflexion where f ′′(x) is not defined.			

Topic/unit (as identified in the IB Diploma subject guide)	Content	Clarification	Allocated time	Assessment	Resources
Calculus **6.4 Integration**	Indefinite integration as anti-differentiation. Indefinite integral of x^n ($n \in Q$) ,, $\frac{1}{x}$ and e^x. The composites of any of these with the linear function ax +b. Integration by inspection, or substitution.		May-June 5 weeks	Assignment	Buchanan, Fensom, Kemp "Mathematics Standard Level, Course companion"- Oxford
6.4 Definite integrals	Anti-differentiation with a boundary condition to determine the constant term. Definite integrals, both analytically and using technology. Areas under curves (between the curve and the x-axis). Areas between curves. Volumes of revolution about the x-axis	The value of some definite integrals can only be found using technology. Students are expected to first write a correct expression before calculating the area. Technology may be used to enhance understanding of area and volume.		Test : Paper1: non GDC Paper 2:GDC	Smedley and Wiseman "Mathematics Standard Level for the IB Diploma"- Oxford IB Questionbank
6.6 Kinematics	Kinematic problems involving displacement (s), velocity (v) and acceleration (a). Total distance travelled.				

Topic/unit (as identified in the IB Diploma subject guide)	Content	Clarification	Allocated time	Assessment	Resources
Internal examinations	Exam preparation and internal examinations		June 1 week		
Internal assessment: **Introduction to the *Exploration***			June 1 week		
Calculus- Revision **6.1,6.2,6.3 Differentiation** **6.4,6.5,6.6 Integration**	Derivative of sin x , cos x , tan x. Differentiation of a sum and a real multiple of these functions. The chain rule for composite trigonometric functions. The product and quotient rules. Local maximum and minimum points. Points of inflexion with zero and non-zero gradients. Indefinite integration as anti-differentiation. Indefinite integral of, sin x , cos x . Integration by inspection, or substitution. Areas between curves. Volumes of revolution about the x-axis.	The value of some definite integrals can only be found using technology. Students are expected to first write a correct expression before calculating the area. Technology may be used to enhance understanding of area and volume.	September-October 3 weeks	Test : Paper1: non GDC Paper 2:GDC	

Topic/unit (as identified in the IB Diploma subject guide)	Content	Clarification	Allocated time	Assessment	Resources
Statistics **5.1 Population, samples, presentation of data**	Concepts of population, sample, random sample, discrete and continuous data. Presentation of data: frequency distributions (tables); frequency histograms with equal class intervals; box-and-whisker plots; outliers. Grouped data: use of mid-interval values for calculations; interval width; upper and lower interval boundaries; modal class.	Continuous and discrete data. Outlier is defined as more than 1.5 IQR × from the nearest quartile. Technology may be used to produce histograms and box-and-whisker plots. Not required: frequency density histograms.	October 3 weeks	Test : Paper1: non GDC Paper 2:GDC	Buchanan, Fensom, Kemp “Mathematics Standard Level, Course companion”- Oxford
5.2 Statistical measures and their interpretations	Central tendency: mean, median, mode. Quartiles, percentiles. Dispersion: range, interquartile range, variance, standard deviation. Effect of constant changes to the original data. Appplications.	On examination papers, data will be treated as the population. Calculation of mean using formula and technology. Students should use mid-interval values to estimate the mean of grouped data. Calculation of standard deviation/variance using only technology. Link to 2.3, transformations.			

Topic/unit (as identified in the IB Diploma subject guide)	Content	Clarification	Allocated time	Assessment	Resources
5.3 Cumulative frequency **5.4 Linear correlation of bivariate data**	Cumulative frequency; cumulative frequency graphs; use to find median, quartiles, percentiles. Linear correlation Pearson's product–moment correlation coefficient r. Scatter diagrams; lines of best fit. Equation of the regression line of y on x. Use of the equation for prediction purposes. Mathematical and contextual interpretation.	Values of the median and quartiles produced by technology may be different from those obtained from a cumulative frequency graph. Independent variable x, dependent variable y. Technology should be used to calculate r. However, hand calculations of r may enhance understanding. Positive, zero, negative; strong, weak, no correlation. The line of best fit passes through the mean point. Technology should be used find the equation. Interpolation, extrapolation. Not required: the coefficient of determination R^2.			
Probability					

Topic/unit (as identified in the IB Diploma subject guide)	Content	Clarification	Allocated time	Assessment	Resources
5.5 Vocabulary **5.6 Combined events** **5.7 Discrete random variables**	Concepts of trial, outcome, equally likely outcomes, sample space (U) and event. The probability of an event A is $P(A) = \frac{n(A)}{n(U)}$. The complementary events A and A' (not A). Use of Venn diagrams, tree diagrams and tables of outcomes. Combined events, $P(A \cup B)$. Mutually exclusive events: $P(A \cap B) = 0$. Conditional probability; the definition $P(A\|B) = \frac{P(A \cap B)}{P(B)}$. Independent events; the definition $P(A\|B) = P(A) = P(A\|B')$ Probabilities with and without replacement. Concept of discrete random variables and their probability distributions. Expected value (mean), E(X) for discrete data. Applications.	The sample space can be represented diagrammatically in many ways. Experiments using coins, dice, cards and so on, can enhance understanding of the distinction between (experimental) relative frequency and (theoretical) probability. Simulations may be used to enhance this topic. Links to 5.1, frequency; 5.3, cumulative frequency. The non-exclusivity of "or". Problems are often best solved with the aid of a Venn diagram or tree diagram, without explicit use of formulae. E(X) =0 indicates a fair game where X represents the gain of one of the players. Examples include games of chance. Link to 1.3, binomial theorem.	November 6 weeks		Buchanan, Fensom, Kemp "Mathematics Standard Level, Course companion"- Oxford Smedley and Wiseman "Mathematics Standard Level for the IB Diploma"- Oxford IB Questionbank

Topic/unit (as identified in the IB Diploma subject guide)	Content	Clarification	Allocated time	Assessment	Resources
5.8 Binomial distribution **5.9 Normal distribution**	Mean and variance of the binomial Distribution. Normal distributions and curves. Standardization of normal variables (z-values, z-scores). Properties of the normal distribution.	Conditions under which random variables have this distribution. Technology is usually the best way of calculating binomial probabilities. Not required: formal proof of mean and variance. Probabilities and values of the variable must be found using technology. Link to 2.3, transformations. The standardized value (z) gives the number of standard deviations from the mean.			
Internal assessment: **Work on the *Exploration***			November 1 week	First draft due in November	
Vectors **4.1 Vectors as displacements in the**	Vectors as displacements in the plane and in three dimensions. Components of a vector; column representation;	Link for three-dimensional geometry, *x,y* and *z*-axes. Components are with respect to the unit vectors i, j and k	December 4 weeks	Assignment	Buchanan, Fensom, Kemp “Mathematics Standard Level, Course

Topic/unit (as identified in the IB Diploma subject guide)	Content	Clarification	Allocated time	Assessment	Resources
plane and in three dimensions.	Algebraic and geometric approaches to the following: • the sum and difference of two vectors; the zero vector, the vector −**v** ; • multiplication by a scalar; parallel vectors; • magnitude of a vector; •unit vectors; base vectors**; i, j** and **k**; • position vectors ; •$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ =***b – a.***	(standard basis). Applications to simple geometric figures are essential. The difference of **v** and **w** is ***v- w*** = ***v + (-w).*** Vector sums and differences can be represented by the diagonals of a parallelogram. Multiplication by a scalar can be illustrated by enlargement. Distance between points A and B is the magnitude of $\overrightarrow{AB}$.		Test: Paper1: non GDC Paper 2:GDC	companion”- Oxford Smedley and Wiseman “Mathematics Standard Level for the IB Diploma”- Oxford IB Questionbank
4.2 The scalar product	T he scalar product of two vectors. Perpendicular vectors; parallel vectors.	The scalar product is also known as the “dot product”. Link to 3.6, cosine rule. For non-zero vectors, ***v · w***= 0 is equivalent to the vectors being perpendicular.			

Topic/unit (as identified in the IB Diploma subject guide)	Content	Clarification	Allocated time	Assessment	Resources
4.3 Vector equation of a line **4.4 Intersection of lines**	The angle between two vectors. Vector equation of a line in two and three dimensions: $\boldsymbol{r}=\boldsymbol{a}+t\boldsymbol{b}$. The angle between two lines. Distinguishing between coincident and parallel lines. Finding the point of intersection of two lines. Determining whether two lines intersect.	Relevance of ***a*** (position) and ***b*** (direction). Interpretation of t as time and ***b*** as velocity.			
Mock Examinations	Exam preparation and examination feedback.		January 3 weeks		
Review	**Examination Preparation**		February-April 7 weeks		

ToK Concepts Covered in IB Diploma Mathematics SL

Topic	ToK Concepts
Algebra	How did Gauss add up integers from 1 to 100? Discuss the idea of mathematical intuition as the basis for formal proof. What is Zeno's dichotomy paradox? How far can mathematical facts be from intuition? Are logarithms an invention or discovery?
Functions and equations	Is mathematics a formal language?
Circular functions and trigonometry	Which is a better measure of angle: radian or degree? What are the "best" criteria by which to decide? Trigonometry was developed by successive civilizations and cultures. How is mathematical knowledge considered from a sociocultural perspective?
Statistics and probability	How easy is it to lie with statistics? Can all data be modeled by a mathematical function? To what extent does mathematics offer models of real life? Is there always a function to model data behavior?

Global Understanding in IB Diploma Mathematics SL

At ISL, global understanding is articulated via the following concepts: Conflict Resolution, Social Justice, Values and Perceptions, Sustainable Development, Interdependence, Human Rights, Diversity and Perspectives. Three levels of depth are considered.

- Understanding and appreciation of their own identity
- Understanding and appreciation of the perspectives of others and our interconnectedness
- Promotion of positive intercultural exchange and/or global interaction

Topic	Global Understanding Concept
Algebra	**Diversity** These concepts are addressed in mathematics SL through the consideration of how the foundations of modern mathematics were laid down many centuries ago by Arabic, Greek, Indian and Chinese civilizations, among others. No culture has exclusive “ownership” of mathematical concepts. Example: The chess legend (Sissa ibn Dahir), The so-called Pascal’s triangle” was known in China much earlier than Pascal.
Circular functions and trigonometry	**Diversity** This concept is addressed in mathematics SL through the development of functions, René Descartes (France), Gotfried Wilhelm Leibniz (Germany) and Leonhard Euler (Switzerland). Why are there 360 degres in a complete turn? Links to Babylonian mathematics.
Statistics and probalility	**Perceptions** This concept is addressed in mathematics SL through the consideration of the benefits of sharing and analyzing data in geography or biology to show patterns and changes. Consideration is also given to the source, quality and availability of some types of data and its effect on the validity of the conclusions that can be drawn. They will also consider if correlation means causation.
Calculus	**Perspectives** This concept is addressed in mathematics SL through the consideration of the “Calculus Controversy”, an argument between 17th-century mathematicians Isaac Newton and Gottfried Leibniz over who had first invented the mathematical study of change, calculus. It is a question that had been the cause of a major intellectual controversy, one that began simmering in 1699 and broke out in full force in 1711.

Development of the IB learner profile in IB Diploma Mathematics SL

Topic	Learner profile attribute	Contribution to the development of the attribute(s) of the IB learner profile
All topics	Communicators	Mathematics SL students are shown though the links made with other subjects, how mathematics is a language in itself that is central to the understanding of events that occur in our world.

	Reflective	They are taught to interpret and express problems using various mathematical forms, both on paper and using technology (including diagrams, graphs and many different forms of notation) . Mathematics SL students are encouraged to regularly consider the reasonableness of their responses to problems in terms of its context and also consider the relevance of their response to other areas of their knowledge. By considering the interrelationships between what they are learning in this subject and their other IB subjects, they will further see the relevance of mathematics to their own lives.

Development of Approaches to Learning in IB Diploma Mathematics SL

Topic	ATL Skill Category (*Communication/ Social/ Self Management/ Research/ Thinking*)	Activities that support the development of these skills
Internal assessment	Communication/Reflection	The emphasis is on using mathematical communication (including formulae, diagrams, graphs and so on), with accompanying commentary, good mathematical writing and thoughtful reflection.
Calculus	Thinking	Throughout the course, particularly in the latter stages of this unit, students are exposed to challenging problems that develop logical, critical and creative thinking, and patience and persistence in problem-solving.