Diploma Programme subject outline—Group 5: mathematics			
School name	International School of Lausanne	School code	001079
Name of the DP subject	Mathematics		
Level	✓ Higher Level	☐ Standard Level	

Course Background

Mathematics is in a sense an international language, and, apart from slightly differing notation, mathematicians from around the world can communicate within their field. Mathematics transcends politics, religion and nationality, yet throughout history great civilizations owe their success in part to their mathematicians being able to create and maintain complex social and architectural structures.

This course caters for students with a good background in mathematics who are competent in a range of analytical and technical skills. The majority of these students will be expecting to include mathematics as a major component of their university studies, either as a subject in its own right or within courses such as physics, engineering and technology. Others may take this subject because they have a strong interest in mathematics and enjoy meeting its challenges and engaging with its problems.

The course focuses on developing important mathematical concepts in a comprehensible, coherent and rigorous way. This is achieved by means of a carefully balanced approach. Students are encouraged to apply their mathematical knowledge to solve problems set in a variety of meaningful contexts. Development of each topic should feature justification and proof of results. Students embarking on this course should expect to develop insight into mathematical form and structure, and should be intellectually equipped to appreciate the links between concepts in different topic areas. They should also be encouraged to develop the skills needed to continue their mathematical growth in other learning environments.

The internally assessed component, the exploration, offers students the opportunity for developing independence in their mathematical learning. Students are encouraged to take a considered approach to various mathematical activities and to explore different mathematical ideas. The exploration also allows students to work without the time constraints of a written examination and to develop the skills they need for communicating mathematical ideas.

This course is a demanding one, requiring students to study a broad range of mathematical topics through a number of different approaches and to varying degrees of depth. Students wishing to study mathematics in a less rigorous environment should therefore opt for one of the standard level courses, mathematics SL or mathematical studies SL. ISL does not offer further mathematics.

Assessment in IB Diploma Mathematics HL

Assessment Components – Higher Level	Weighting (%)
External assessment (5 hours)	
Paper 1 (2 hours) This paper consists of section A, short-response questions, and section B, extended-response questions. Questions of varying levels of difficulty and length are set and knowledge of **all** core topics is required for this paper. The emphasis is on problem-solving. Students are not permitted access to any calculator on this paper.	30%
Paper 2 (2 hours) This paper consists of section A, short-response questions, and section B, extended-response questions. Questions of varying levels of difficulty and length are set and knowledge of all core topics is required for this paper. The emphasis is on problem-solving. A GDC is required for this paper, but not every question will necessarily require its use.	30%
Paper 3 (1 hour) HL option paper (60 marks). This paper consists of a small number of compulsory extended-response questions based on the option chosen. The emphasis is on problem-solving. GDC calculators may be used.	20%
Internal Assessment	
An individual exploration.	20%

The Nature and Timing of Internal Assessment in IB Diploma Mathematics HL

The exploration is a piece of written work that involves investigating an area of mathematics. It enables students to demonstrate the application of their mathematical skills and knowledge, and to pursue their personal interests, without the time limitations and other constraints that are associated with written examinations. The product is a short report written by the student based on a topic chosen by him or her, and it should focus on the mathematics of that particular area. The emphasis is on mathematical communication (including formulae, diagrams, graphs and so on), with accompanying commentary, good mathematical writing and thoughtful reflection.

Internal Assessment in IB Diploma mathematics HL is undertaken in the first semester of Year 13. A full first draft is due in mid-November, and the final version in the second week of December (see calendar of deadlines).

Topic/unit (as identified in the IB Diploma subject guide)	Content	Clarification	Allocated time	Assessment	Resources
YEAR 12 **1. Algebra**	The aim of this topic is to introduce students to some basic algebraic concepts and applications.		September 5 weeks		
1.1 Sequences and series	Arithmetic sequences and series; sum of finite arithmetic series; geometric sequences and series; sum of finite and infinite geometric series. Sigma notation. Applications.	Include recursive definitions. Link infinite geometric series with limits of convergence 6.1. Applications to compound interest and population growth.			
1.2 Exponents and logarithms.	Laws of exponents; laws of logarithms. Change of base rule.	Exponents and logarithms are developed further in 2.4.			
1.3 Counting principles	Factorials. Rules of factorials. Permutations and combinations, using both the formulae and technology. The binomial theorem: expansion of $(a+b)^n, n \in \mathbb{N}$	 Link to 5.4. Link to 5.6, the binomial distribution. Not required: Permutations where some objects are identical. Circular arrangements. Proof of binomial theorem.			
1.4 Proof by mathematical induction.	Links to a wide variety of topics, including complex numbers, nth derivatives, sums of series, divisibility proofs. De Moivre's theorem to be done after 1.7.	Necessary steps for a rigorous proof and common errors to be avoided. Link with 1.7		Unit Assessment	

Topic/unit (as identified in the IB Diploma subject guide)	Content	Clarification	Allocated time	Assessment	Resources
1.5 Complex numbers.	The number i as the square root of -1; real and imaginary parts of a complex number; conjugate; modulus and argument. Cartesian form $z = a + ib$ Sums, products, quotients, square roots of complex numbers in Cartesian form.	When solving problems, students may need to use technology. Familiarity with complex number functions of GDC is essential.	October 3 weeks		
1.8 Conjugate roots	Complex roots of polynomial equations with real coefficients occur in conjugate pairs.	Link with the fundamental theorem of algebra 2.5 and 2.7.		Unit Assessment	
2. Functions and equations	The aims of this topic are to explore the notion of function as a unifying theme in mathematics, and to apply functional methods to a variety of mathematical situations		November 3 weeks		
2.1 Basic functions.	Concept of a function: domain, range; image (value). Odd and even functions. Composite functions. Identity function. One-to-one and many-to-one functions. The inverse function, including domain restriction. Self-inverse functions.	 Links with 6.2, 8.8 Links with 3.4, 8.3 Links with 6.2, 8.8			
2.2 The graph of a function and its equation.	Investigation of key features of graphs, such as maximum and minimum values, intercepts, horizontal and vertical asymptotes and symmetry, and consideration of domain and range. The graphs of the functions $y = f(\|x\|)$ and $y = \|f(x)\|$. The graph of the reciprocal function, given the graph of the function.	Use of technology to graph and understand the characteristics of a variety of functions.			
2.3 Transformations of graphs.	Transformations of graphs: translations; stretches; reflections in the axes.	Link to 3.4.			

Topic/unit (as identified in the IB Diploma subject guide)	Content	Clarification	Allocated time	Assessment	Resources
	The graph of the inverse function as a reflection in $y = x$.	Effect of a transformation on both the algebraic expression and the graph of a function.			
2.4 Rational functions, exponential functions, logarithmic functions.	The rational function $f(x) = \frac{ax+b}{cx+d}$ and its graph. The function $f(x) = a^x, a > 0$, and its graph. The function $f(x) = log_a x, x > 0,$ and its graph	The reciprocal function is a particular case. Graphs should include any asymptotes and any intercepts with axes. Exponential and logarithmic functions as inverses of each other. Link to 6.2 and the significance of e. Application of concepts in 2.1, 2.2 and 2.3.			
2.5 Polynomials functions.	Polynomials and their graphs. The factor theorem. The remainder theorem. The fundamental theorem of algebra.	The graphical significance of repeated factors. The relationship between the degree of a polynomial function and the possible number of x-intercepts.			
2.6 Solutions to equations.	Solving quadratic equations using the quadratic formula. Use of the discriminant to determine the nature of the roots.	Interchangeable use of solution or roots of equations, and zeros of functions.			

Topic/unit (as identified in the IB Diploma subject guide)	Content	Clarification	Allocated time	Assessment	Resources
	Solving polynomial equations both graphically and algebraically. Sum and product of the roots of polynomial equations. Solution of $a^x = b$ using logarithms. Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach.	Link the solution of polynomial equations to conjugate roots in 1.8. For the polynomial equation $\sum_0^n a_r x^r$, the sum of the roots is $\frac{-a_{n-1}}{a_n}$ and the product of the roots is $\frac{(-1)^n a_o}{a_n}$ Graphing techniques and use of GDC solve function.			
2.7 Solutions of $g(x) \geq f(x)$.	Graphical or algebraic methods for simple polynomials up to degree 3. Use of technology for these and other functions.			Unit Assessment	
3. Circular functions and trigonometry.	The aims of this topic are to explore the circular functions, to introduce some important trigonometric identities and to solve triangles using trigonometry.		November December 4 weeks		
3.1 Radian measure and use of radians in the circle.	Radians and degrees. Length of an arc, area of a sector.	On exam papers, radian measure should be assumed unless indicated otherwise. Segments and triangles.			

Topic/unit (as identified in the IB Diploma subject guide)	Content	Clarification	Allocated time	Assessment	Resources
3.2 The unit circle and the six trigonometric ratios.	Definition of $\cos\theta, \sin\theta, \tan\theta$ in terms of the unit circle. Exact values of the sine, cosine and tangent of the special triangle angles, and their multiples. Definition of the reciprocal trigonometric ratios $\sec\theta, \csc\theta, \cot\theta$. The three Pythagorean identities.				
3.3 Trigonometric identities.	Compound angle identities. Double angle identities.	Proof of compound angle identities not required. Derivation of double angle identities from compound angle identities. Finding possible values of trigonometric ratios without finding θ, for example, finding $\sin 2\theta$ given $\sin\theta$.			
3.4 Composite functions.	Composite functions of the form $f(x) = a\sin(b(x + c)) + d$ Applications.	amplitude – brequency (or beriod) –corizontal – dertical Link with 2.3 transformations of graphs.			
3.5 Inverse trigonometric functions.	The inverse functions: arcsine, arccosine, and arctangent; their domains and ranges; their graphs.				

Topic/unit (as identified in the IB Diploma subject guide)	Content	Clarification	Allocated time	Assessment	Resources
3.6 Solving trigonometric equations.	Algebraic and graphical methods of solving trigonometric equations in a finite interval, including the use of trigonometric identities and factorization.	The general solution of trig equations is not required.			
3.7 Triangle trigonometry.	The cosine rule. The sine rule, including the ambiguous case. The area of a triangle.	Applications including navigation problems, geometry in two and three dimensions, angles of elevation and depression.		Unit Assessment	
Internal Examinations	Exam preparation, internal assessment and examination feedback.		January 2 weeks		
1. Further complex numbers			January 2 weeks		
1.6 Modulus–argument (polar) form.	$z = r(cos\theta + isin\theta) = rcis\theta = re^{i\theta}$ The complex plane; Argand diagrams.	$re^{i\theta}$ is also known as Euler's form. Converting between different forms of complex numbers. $e^{i\theta} + 1 = 0$			
1.7 Powers and roots of complex numbers.	De Moivre's theorem and problem solving applications; *n*th roots of a complex number. Proof of de Moivre's theorem using mathematical induction.	Use of the Argand plane to illustrate powers and roots of complex numbers.		Unit Assessment	
6. Calculus.	The aim of this topic is to introduce students to the basic concepts and techniques of differential and integral calculus and their application.		February March 5 weeks		
6.1 Introductory calculus.	Informal ideas of limit, continuity and convergence. Definition of derivative from first principles.	Include result $\lim_{\theta \to 0} \left(\frac{sin\theta}{\theta}\right) = 1$			

Topic/unit (as identified in the IB Diploma subject guide)	Content	Clarification	Allocated time	Assessment	Resources
	The derivative interpreted as a gradient function and as a rate of change. Finding equations of tangents and normals. Identifying increasing and decreasing functions. The second derivative. Higher derivatives.	Link to series in 1.1. Use of this definition for polynomials only. Link to binomial theorem in 1.3. Liebnitz's notation and Lagrange's notation for differentiation. Use of both algebra and technology. Link with induction in 1.4.			
6.2 Differentiation.	Derivatives of x^n, $\sin x$, $\cos x$, $\tan x$, e^x and $\ln x$. Differentiation of sums and multiples of functions. The product and quotient rules. The chain rule for composite functions. Related rates of change. Implicit differentiation. Derivatives of $\sec x$, $\csc x$, $\cot x$, a^x, $\log_a x$, $\arcsin x$, $\arccos x$ and $\arctan x$				
6.3 Uses of differentiation.	Local maximum and minimum values. Optimization problems.	Testing for the maximum or minimum using the change of sign of the first derivative		Unit Assessment	

Topic/unit (as identified in the IB Diploma subject guide)	Content	Clarification	Allocated time	Assessment	Resources
	Points of inflexion with zero and non-zero gradients. Graphical behaviour of functions, including the relationship between the graphs of f, f' and f''. Not required: points of inflexion, where $f''(x)$ is not defined. For example, $y = x^{1/3}$ at (0, 0).	and using the sign of the second derivative. Use of the terms "concave up" for $f''(x) > 0$, "concave down" for $f''(x) < 0$ At a point of inflexion, $f''(x) = 0$ and changes sign (concavity change).			
6. Further Calculus					
6.4 Integration.	Indefinite integration as anti-differentiation. Indefinite integral of x^n, $\sin x$, $\cos x$, e^x. Other indefinite integrals using the results from 6.2. The composites of any of these with a linear function.	Indefinite integral interpreted as a family of curves. $\int \frac{1}{x} dx = ln\|x\| + c$ Examples should include $\int (2x-1)^5 dx$, $\int \frac{1}{3x+4} dx$, $\int \frac{1}{x^2+2x+5} dx$	April May 5 weeks		
6.5 Definite integrals.	Anti-differentiation with a boundary condition to determine the constant of integration. Definite integrals. Area of the region enclosed by a curve and the x-axis or y-axis in a given interval; areas of regions enclosed by curves.	The value of some definite integrals can only be found using technology**.**			

Topic/unit (as identified in the IB Diploma subject guide)	Content	Clarification	Allocated time	Assessment	Resources
	Volumes of revolution about the x-axis or y-axis.				
6.6 Kinematics.	Kinematic problems involving displacement s, velocity v and acceleration a. Total distance travelled.	$\boldsymbol{v} = \frac{ds}{dt}$ $a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = v\frac{dv}{ds}$ $\int_{t_1}^{t_2} \lvert v \rvert dt$			
6.7 Harder integration.	Integration by substitution. Integration by parts.	On examination papers, non-standard substitutions will be provided. Link to 6.2. $\int x \sin x \, dx$ $\int \ln x \, dx$ Repeated integration by parts; e.g. $\int x^2 e^x dx$ $\int e^x \sin x \, dx$		Unit Assessment	
Internal Examinations	Exam preparation, internal assessment and examination feedback.		May June 4 weeks		
The Exploration and Vectors.	Students begin preparing their internal assessment and begin the Vectors unit		June 3 weeks		
YEAR 13					

Topic/unit (as identified in the IB Diploma subject guide)	Content	Clarification	Allocated time	Assessment	Resources
4. Vectors.	The aim of this topic is to introduce the use of vectors in two and three dimensions, and to facilitate solving problems involving points, lines and planes.		June September 4 weeks		
4.1 Basic vectors.	The concept of a vector. Representation of vectors using directed line segments. Unit vectors; base vectors **i, j, k**. Components of a vector: as a column vector; using **i, j, k** components. Algebraic and geometric approaches to the following: • the sum and difference of two vectors; • the zero vector **0**, the vector −**v** ; • multiplication by a scalar, *k***v** ; • magnitude of a vector; • position vectors	Proofs of geometrical properties using vectors. The distance between the points A and B is the magnitude of the vector joining the points.			
4.2 Scalar product.	Definition of scalar product. Properties of scalar product. The angle between two vectors. Perpendicular vectors. Parallel vectors.	For non-zero vectors, $\mathbf{v}.\mathbf{w} = 0 \Leftrightarrow$ vectors are perpendicular. Parallel $\Leftrightarrow |\mathbf{v}.\mathbf{w}| = |\mathbf{v}||\mathbf{w}|$ Link to 3.6.			
4.3 Vector equation of a line in two and three dimensions.	Different forms of the equation of a line. Simple applications to kinematics. The angle between two lines.	Vector form using column vectors. Vector form using component vectors.			

Topic/unit (as identified in the IB Diploma subject guide)	Content	Clarification	Allocated time	Assessment	Resources
		Parametric form. Cartesian form.			
4.4 Two lines.	Coincident, parallel, intersecting and skew lines; distinguishing between these cases. Points of intersection of two lines.				
4.5 Vector product.	Definition of vector product. Properties of vector product. Vector product in terms of **v**, **w**, sinθ and **n**, where **n** is the unit normal vector whose direction is given by right-hand screw rule. Geometric interpretation of the magnitude of the vector product. $\lvert\mathbf{v}\times\mathbf{w}\rvert$.	$\mathbf{v}\times\mathbf{w}=-\mathbf{w}\times\mathbf{v}$ $\mathbf{u}\times(\mathbf{v}+\mathbf{w})=$ $\mathbf{u}\times\mathbf{v}+\mathbf{u}\times\mathbf{w}$ $(k\mathbf{v})\times\mathbf{w}=k(\mathbf{v}\times\mathbf{w})$ $\mathbf{v}\times\mathbf{v}=\mathbf{0}$ Areas of triangles and parallelograms.			
4.6 Vector equation of a plane..	Different forms of the equation of a plane. Being able to convert between the different forms of the equation of a plane.	Vector form using column vectors. Vector form using component vectors. Use of a normal vector to obtain the form $\mathbf{r}.\mathbf{n}=\mathbf{a}.\mathbf{n}$ Cartesian form $ax+by+cz=d$			
4.7 Three planes.	Intersections of: a line with a plane; two planes; three planes. Angle between: a line and a plane; two planes.	Link to systems of equations in 1.9. Geometrical interpretation of solutions.			

Topic/unit (as identified in the IB Diploma subject guide)	Content	Clarification	Allocated time	Assessment	Resources
1.9 Solutions of systems of linear equations.	A maximum of three equations in three unknowns, including cases where there is a unique solution, an infinity of solutions or no solution. Consistent and inconsistent systems. Row reduction. The general solution for a system of equations that has an infinity of solutions.	Link to vectors in 4.7 during vectors unit. Both algebraic and technological methods necessary. Matrix methods are useful here but would need to be taught as a separate unit.		Unit Assessment	
5. Statistics and probability.	The aim of this topic is to introduce basic concepts of statistics and probability. It is expected that most of the calculations required will be done on a GDC. The emphasis is on understanding and interpreting the results obtained. Statistical tables are not used.		October November 6 weeks		
5.1 Manipulation and presentation of statistical data.	Concepts of population, sample, random sample and frequency distribution of discrete and continuous data. Grouped data: mid-interval values, interval width, upper and lower interval boundaries. Mean, variance, standard deviation.	Not required: estimation of mean and variance of a population from a sample. Data will be treated as the population. Familiarity with 'sigma formulae' for mean and variance.			
5.2 The laws of probability.	Concepts of trial, outcome, equally likely outcomes, sample space and event.				

Topic/unit (as identified in the IB Diploma subject guide)	Content	Clarification	Allocated time	Assessment	Resources
	Probability of an event. Complementary events A and A′ (not A). Use of Venn diagrams, tree diagrams, counting principles and tables of outcomes to solve problems.				
5.3 Combined events.	The formula for P(A ∪ B). Mutually exclusive events.				
5.4 Conditional probability.	Definition of conditional probability. Independent events and conditional probability: P(A|B) = P(A) = P(A|B ′) The use of Bayes' Theorem for a maximum of three events.	Use of P(A ∩ B) = P(A)P(B) to show independence.			
5.6 Binomial and Poisson distributions.	Binomial distribution, its mean and variance. Poisson distribution, its mean and variance.	Link to binomial theorem 1.3 Conditions under which random variables have these distributions. Formal proofs of means and variances are not required.			
5.7 Normal distribution.	Properties of the normal distribution. Standardization of normal variables.	Probabilities and values of the variable must be found using technology. The standardized value (z) gives the number of		Unit Assessment	

Topic/unit (as identified in the IB Diploma subject guide)	Content	Clarification	Allocated time	Assessment	Resources
		standard deviations from the mean. Link to transformations of graphs in 2.3.			
7. Option:	Students will study one of the following options: Statistics and probability Discrete Mathematics Sets relations Groups Calculus		November December January 8 weeks		

ToK Concepts Covered in IB Diploma Mathematics HL

Topic	ToK Concepts
1.2 Logarithms	The nature of mathematics and science. Were logarithms an invention or a discovery? Mathematics is a puzzling phenomenon. A fundamental question for all knowers is whether mathematical knowledge really exists independently of our thinking about it. Is it there "waiting to be discovered" or is it a human creation?
1.3 Counting principles	The nature of mathematics. The unforeseen links between Pascal's triangle, counting methods and the coefficients of polynomials. Is there an underlying truth that can be found linking these?
1.4 Proof	Knowledge claims in mathematics. Do proofs provide us with completely certain knowledge?
1.5 Complex numbers	Mathematics and the knower. Do the words imaginary and complex make the concepts more difficult than if they had different names?
1.6 Complex numbers	Mathematics and the knower. Why might it be said that $e^{i\pi} + 1 = 0$ is beautiful?
3.2 The unit circle and the six trigonometric ratios	Mathematics and knowledge claims. If trigonometry is based on right triangles, how can we sensibly consider trigonometric ratios of angles greater than a right angle?
4.7 Three planes	Mathematics and the knower. Why are symbolic representations of three-dimensional objects easier to deal with than visual representations? What does this tell us about our knowledge of mathematics in other dimensions?

Topic	ToK Concepts
5.7 The normal distribution	Mathematics and knowledge claims. To what extent can we trust mathematical models such as the normal distribution?
6.1 Introductory calculus	The nature of mathematics. Does the fact that Leibniz and Newton came across the calculus at similar times support the argument that mathematics exists prior to its discovery?
7.1 Sets	Russell's paradox.

Global Understanding in IB Diploma Mathematics HL

At ISL, global understanding is articulated via the following concepts: Conflict Resolution, Social Justice, Values and Perceptions, Sustainable Development, Interdependence, Human Rights, Diversity and Perspectives. Three levels of depth are considered.

- Understanding and appreciation of their own identity
- Understanding and appreciation of the perspectives of others and our interconnectedness
- Promotion of positive intercultural exchange and/or global interaction

Topic	Global Understanding Concept
1. Algebra	**Diversity** The properties of Pascal's triangle were known in a number of different cultures long before Pascal (e.g. the Chinese mathematician Yang Hui).
3. Circular functions and trigonometry	**Perspectives** Mathematics is an international language, and, apart from slightly differing notation, mathematicians from around the world can communicate within their field. Mathematics transcends politics, religion and nationality, yet throughout history great civilizations owe their success in part to their mathematicians being able to create and maintain complex social and architectural structures. Discuss the origin of degrees in the mathematics of Mesopotamia and why we use minutes and seconds for time.
5. Statistics and probability	**Perceptions** De Moivre's derivation of the normal distribution and Quetelet's use of it to describe *l'homme moyen*.
6. Calculus	**Perspectives** This concept is addressed in mathematics through the consideration of the "Calculus Controversy", an argument between 17th-century mathematicians Isaac Newton and Gottfried Leibniz over who had first invented the mathematical study of change, calculus. It is a question that had been the cause of a major intellectual controversy, one that began simmering in 1699 and broke out in full force in 1711. Illustrating the characters and personalities of the mathematicians concerned and the historical context in which they worked brings home the human and cultural dimension of mathematics.

Development of the IB learner profile in IB Diploma Mathematics HL

Topic	Learner profile attribute	Contribution to the development of the attribute(s) of the IB learner profile
All topics All topics	Communicator Risk taker	One of the aims of the HL course is to enable students to communicate clearly and confidently in a variety of ontexts. Students are expected to transform common realistic contexts into mathematics; comment on the context; sketch or draw mathematical diagrams, graphs or constructions both on paper and using technology; record methods, solutions and conclusions using standardized notation. In the HL course, students investigate unfamiliar situations, both abstract and real-world, involving organizing and analysing information, making conjectures, drawing conclusions and testing their validity. Students are encouraged to learn by experimentation, questioning and discovery, by being active participants in learning activities rather than recipients of instruction.

Development of Approaches to Learning in IB Diploma Mathematics HL

Topic	ATL Skill Category (*Communication/ Social/ Self Management/ Research/ Thinking*)	Activities that support the development of these skills
Topics 1 and 7 Algebra and Sets, Relations and Groups	Thinking	Students are presented with problems that encourage them to employ and refine their powers of abstraction and generalization. They learn to construct mathematical arguments through use of precise statements, logical deduction and inference, and by the manipulation of mathematical expressions.
Calculus	Thinking	Throughout the course, particularly in the latter stages of this unit, students are exposed to challenging problems that develop logical, critical and creative thinking, and patience and persistence in problem-solving.